COMM 215: Business Statistics Solution to Practice Problems 2

Simple Linear Regression and Correlation

1 a) $\hat{y}=1.4235+0.53x$

b) $R^2=0821$. That is, 81.2% of the variation in ABC's rate of return is explained by the market of return.

c) $\begin{cases} H_0:\beta_1=0 \\ H_0:\beta_1\neq 0 \end{cases}$ $t=\dfrac{0.53}{0.103}=5.15$. Since 5.15 > 2.447, reject H_0 and conclude that model is significiant.

d) $\hat{y}=1.4235+0.53(5)=4.07$

$$4.07\pm 3.707(2.8225\sqrt{1+\frac{1}{8}+\frac{(5-2.5)^2}{752}} \Rightarrow 4.07\pm 11.137 \text{ or } (-7.067,\ 15.207)$$

2 a) $\hat{y}=10.548+0.00578x$

For each additional million dollars, the price per share

increases by 0.00578 (in \$) while the initial price (constant) is \$10.55

b) $\begin{cases} H_0:\beta_1=0 \\ H_0:\beta_1\neq 0 \end{cases}$ $t=\dfrac{0.00578}{0.0039}=1.49$

p-value = $p(t \geq 1.49)\times 2 \Rightarrow (.05\times 2) \leq p \leq (.10\times 2)$, or $.10 \leq p \leq .20$

So reject H_0 if $\alpha > .20$, but do not reject H_0 if $\alpha < .10$

c) $R^2=0.219$. That is, 21.9% of the variation in price per share is explained by the size of the offering.

d) $\hat{y}=10.548+0.00578(70)=10.95$. That is, the estimated mean price per share is \$10.95.

The 95% confidence interval estimate of the mean price per share for all companies with a size offering of \$70 million:

$$10.95\pm 2.306(.39)\sqrt{\frac{1}{10}+\frac{(70-64.3)^2}{10155.4}} \text{ or } 10.95\pm 0.29 \text{ or } (10.66,\ 11.24)$$

3 a) $\hat{\beta}_1 = \dfrac{SS_{xy}}{SS_{xx}}$, where

$$SS_{xy} = 21115.07 - \frac{(92.93)(2725)}{12} = 12.2158$$

$$SS_{xx} = 720.22 - \frac{(92.93)^2}{12} = 0.554592$$

$$SS_{yy} = 619207 - \frac{(2725)^2}{12} = 404.917$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{12.2158}{0.554592} = 22.0267$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = \left(\frac{2725}{12}\right) - 22.0267\left(\frac{92.93}{12}\right) = 56.5048$$

$\hat{y} = 56.5048 + 22.0267x$, For each additional one percent increase in interest rates, the futures index increase by 22.0267 points.

b) $\begin{cases} H_0: \beta_1 = 0 \\ H_0: \beta_1 \neq 0 \end{cases}$ reject H_0 if $t > |t_{.005,\,10}| = 3.169$; test stat: $t = \dfrac{\hat{\beta}_1}{S_{\hat{\beta}_1}}$

$$S_{\hat{\beta}_1} = \frac{S}{\sqrt{SS_{xx}}}, \quad S = \sqrt{\frac{(SS_{yy} - \hat{\beta}_1 SS_{xy})}{(n-2)}} = \sqrt{\frac{404.917 - (22.0267)(12.2158)}{10}} = 3.68567$$

$$S_{\hat{\beta}_1} = \frac{3.68567}{\sqrt{0.554592}} = 4.94915, \qquad t = \frac{\hat{\beta}_1}{S_{\hat{\beta}_1}} = \frac{22.0267}{4.94915} = 4.45061$$

Since t=4.5061 > 3.169, reject H_0 at 5% level of significance and conclude that interest rate is a significant predict of futures index.

p value: .001 <p-value < .002

c) $$r = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}} = \frac{12.2158}{\sqrt{0.554592}\sqrt{404.917}} = 0.815180$$

The value of r suggests a strong positive correlation between interest rates and future index.

d) The 95% confidence interval estimate for the meanfutures index when interest rate is 7.8% :

$\hat{y} = 56.5048+22.0267(7.8) =228.313;$

$$228.313 \pm 2.228(3.68567)\sqrt{\frac{1}{12}+\frac{(7.8-7.74417)^2}{0.554592}}$$

$228.313\pm2.228(3.68567)(\ 0.298252)$

228.313 ± 2.44915 or $(225.864,\ 230.762)$